Quaid-I-Azam University
Department of Mathematics
PhD Admission test
Pure Mathematics

Spring 2022 Total time: 90 mins Total marks: 100

Question 1 (a) *Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function and let $A = \{x \in \mathbb{R} : f(x) = 0\}$. If (x_n) is in A and $\lim_{n \to \infty} x_n = x$, then show that $x \in A$.*

(b) *Using sequential criteria, does $\lim_{x \to 0}(x^2 + \operatorname{sgn}(x))$ exist?*

Question 2 *Let $U = \{z \in C : |z| = 1\}$ and $G = \{r \in R : r \neq 0\}$, where C and R are the set of complex and real numbers resp. Then show that $(U, .)$ is not isomorphic to either $(R, +)$ or $(G, .)$.*

Question 3 (a) *Describe completely the characteristic polynomial and minimal polynomial of a linear transformation. Furthermore, when chracteristic roots are eigen values?*

(b) *Over the real field $\mathbf{R}$ consider*

$$A = \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix} \quad and \quad B = \begin{bmatrix} r & s \\ 0 & r \end{bmatrix}, \quad where \ s \neq 0$$

Verify that the characteristic polynomials of A and B coincide while the minimal polynomials are different.

Question 4 *A mapping $f : X \to Y$ from a topological space X into a topological space Y is continuous if and only if the inverse image of every closed set V in Y is closed in X.*

Question 5 (a) *Compute the torsion of the circular helix*

$$r(\theta) = (a \cos(\theta), a \sin(\theta), b\theta).$$

(b) *When $b = 0$, what does $r(\theta)$ in part (a) represent?*