

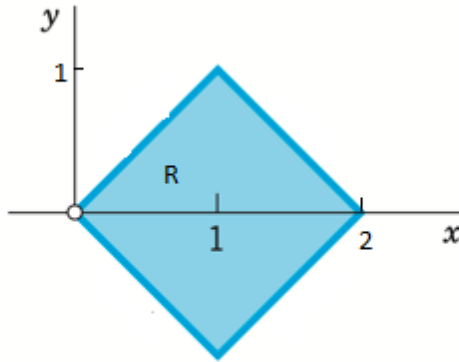
Quaid-I-Azam University
Department of Mathematics
PhD Admission test
Applied Mathematics

Spring 2022 Total time: 90 mins Total marks: 100

Question 1 Evaluate the integral

$$\int \int_R (x^2 + y^2) dx dy,$$

where R is the region shown in the following figure



Question 2 Solve the given initial value problem,

$$\begin{aligned} \frac{\partial v}{\partial t} &= c^2 \frac{\partial^2 v}{\partial z^2}, \\ v(z, 0) &= g(z), \quad (-\infty < z < \infty) \end{aligned} \tag{1}$$

1. Express the solution in terms of Fourier integral.
2. Find the temperature in the infinite bar if the initial temperature $g(z) = A_0$ for $|z| < 1$ and zero elsewhere.

Question 3 Solve the following system using Cholesky's decomposition method:

$$\begin{aligned} 4x_1 + 2x_2 + 14x_3 &= 14 \\ 2x_1 + 17x_2 - 5x_3 &= -101 \\ 14x_1 - 5x_2 + 83x_3 &= 155. \end{aligned}$$

Question 4 Find eigen values and eigen functions of $y'' + \lambda y = 0$, with $y(-1) = y(1)$ and $y'(-1) = y'(1)$.

Question 5 Compute

$$\oint_C \frac{z^2 \sin(z)}{4z^2 - 1} dz$$

where (a) $C : |z| = 1$ (b) $C : |z - i| = 1$ (c) $|z - 1| = 1$