Department of Mathematics, Quaid-i-Azam University, Islamabad
M.Phil Admission Test – Spring 2019

Student's name: ______________________ Time duration: 90 minutes

Note: Attempt all questions.

Q.1) Find cartesian equations of the following parametrised curves.

(a) $\gamma(t) = (2t+1, t)$

(b) $\gamma(t) = (\frac{2t}{t^2+1}, \frac{t^2-1}{t^2+1})$.

Q.2) Find a rational parametrisation of the unit sphere $S^2 = \{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2+z^2 = 1\}$. Explain how this parametrisation can be used to show that S^2 is a two dimensional manifold.

Q.3) Let (X, τ) be a toplogical space and $A \subseteq X$. Show that A is closed if and only if $A = \overline{A}$

Q.4) Solve

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = 0.$$

Q.5) Show that in cylindrical coordinates r, θ, z defined by the relations $x = rcos\theta, y = rsin\theta, z = z$, the Laplace equation takes the form

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial z^2} = 0.$$

Q.6) State and prove Euler's theorem about the general motion of rigid body.

Q.7) Let $n \geq 5$ be an integer. Prove that the alternating group A_n is not abelian.

Q.8) If $A = \{(a,b,c) \mid a+b+c = 0\} \subset \mathbb{R}^3$. Show that A is a subspace of $\mathbb{R}^3$. Also compute its basis.

Q.9) Show that

$$Res_{z=\pi i}\frac{z - sinhz}{z^2 sinhz} = \frac{i}{\pi}.$$

Q.10) Prove that topology induced by a norm space $(X, \|\|)$ is Hausdroff topology.

Q.11) Evaluate

$$lim_{x\to 0}\frac{\sqrt{1+x} - \sqrt{1-x}}{x}$$

Q.12) Evaluate $\int_0^\infty \frac{\sqrt{x}}{x^2+1}dx$.

Q.13) Let $f(x) = cosx - 2x$ and $x_0 = 0.5$. Use Newton's method to find solution accurate to five decimal places. Make sure your calculator is in radian mode.

Good Luck